# Geometry Learning Progression

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## Summary of Year

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

### Recommended Fluencies for Geometry

- Triangle congruence and similarity criteria.
- Using coordinates to establish geometric results.
- Calculating length and angle measures.
- Using geometric representations as a modeling tool.
- Using construction tools, physical and computational, to draft models of geometric phenomenon.

Revised 5.27.15

**Unit 1: Geometric Figures**
**Unit 2: Congruency and Proofs**
**Unit 3: Similarity, Proof, and Trigonometry**
**Unit 4: Conditional Probability**
**Unit 5: Right Triangle Trigonometry & Pythagorean Theorem**
**Unit 6: Extending to Three Dimensions**
**Unit 7: Connecting Algebra and Geometry Through Coordinates**
**Unit 8: Circles with and without Coordinates**

### WSLS (CCSS) Major Emphasis Clusters

**Congruence**

- Understand congruence in terms of rigid motions
- Prove geometric theorems

**Similarity, Right Triangles, and Trigonometry**

- Understand similarity in terms of similarity transformations
- Prove theorems using similarity
- Define trigonometric ratios and solve problems involving right triangles

**Expressing Geometric Properties with Equations**

- Use coordinates to prove simple geometric theorems algebraically

**Modeling with Geometry**

- Apply geometric concepts in modeling situations

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## Rationale for Unit Sequence in Geometry

**Unit 1:** In Geometry, students formalize much of the geometric exploration from middle school. In this unit, students develop rigorous definitions of three familiar congruence transformations: reflections, translations, and rotations and use these transformations to discover and explore geometric properties. Throughout the course, students will use transformations as a tool to analyze and describe relationships between geometric figures. Students will use constructions to develop geometric vocabulary.

**Unit 2**: In this unit, students use their knowledge of rigid motions to prove geometric properties and congruence of triangles and parallelograms. Students will explore, prove and use theorems about triangles. Students will use inductive and deductive reasoning to make conjectures about geometric figures. Students will understand the importance of counterexamples to prove or disprove conjectures.

**Unit 3**: In this unit students will develop an understanding and definition of similarity and scale factor in polygons. They will then use the definition of similarity and dilations (discussed in a previous unit) to prove triangles are similar. Students will use their understanding of similarity to solve problems.

**Unit 4:** In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data, including sample surveys, experiments, and simulations, and the role that randomness and careful design play in the conclusions that can be drawn. Students create theoretical and experimental probability models following the modeling cycle (see page 61 of CCLS). They compute and interpret probabilities from those models for compound events, attending to mutually exclusive events, independent events, and conditional probability.

**Unit 5:** Students will use trigonometric ratios and Pythagorean Theorem to determine side lengths and angles of right triangles and use these skills to solve real world problems. Students will also explain and use the relationship between sine and cosine of complementary angles.

**Unit 6:** Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the

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shapes of cross-sections and the result of rotating a two-dimensional object about a line. They reason abstractly and quantitatively to model problems using volume formulas.

The standard on the volume of a sphere is an extension of G-GMD.1. **Standard G.GMD.1 is not assessed on the WA State Geometry EOC**. It is explained by the teacher in this grade and used by students in G-GMD.3. **Note: Students are not assessed on proving the volume of sphere formula until Precalculus.**

**Unit 7:** Building on their work with the Pythagorean Theorem in 8th grade to find distances, students analyze geometric relationships in the context of a rectangular coordinate system, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Students attend to precision as they connect the geometric and algebraic definitions of parabola. They solve design problems by representing figures in the coordinate plane, and in doing so, they leverage their knowledge from geometry by combining it with the solving power of algebra inherent in analytic geometry.

**Unit 8:** In this unit students prove and apply basic theorems about circles, such as: a tangent line is perpendicular to a radius theorem, the inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students explain the correspondence between the definition of a circle and the equation of a circle written in terms of the distance formula, its radius, and coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations.

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## Alignment Chart

| Unit and Approximate Number of Instructional Days | Washington State Learning Standards Addressed in Geometry Units |
| --- | --- |
| **Unit 1:**<br><br>**Geometric Figures**<br>(21 days)<br><br>**SMP-1,2,3,4,5,6,7,8** | *Experiment with transformations in the plane*<br><br>G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.<br><br>G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).<br><br>G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.<br><br>G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.<br><br>G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.<br><br>*Understand congruence in terms of rigid motions*<br><br>G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. |

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| | *Prove geometric theorems*<br><br>G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*<br><br>G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*<br><br>G-CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. |
| **Unit 2:**<br><br>**Congruency and Proofs**<br>(25 days)<br><br>**SMP-1,2,3,4,5,6,7,8** | *Understand congruence in terms of rigid motions*<br><br>G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.<br><br>G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.<br><br>*Prove geometric theorems*<br><br>G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the* |

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| | *segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*<br><br>G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* |
| **Unit 3:**<br><br>**Similarity, Proof, and Trigonometry**<br>(15 days)<br><br>**SMP-1,2,3,4,5,6,7,8** | *Understand similarity in terms of similarity transformations*<br><br>G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:<br>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.<br>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.<br><br>G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.<br><br>G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.<br><br>*Prove theorems involving similarity*<br><br>G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* |

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| | G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. |
| **Unit 4:**<br><br>**Conditional Probability**<br>(15 days)<br><br>**SMP-1,2,3,4,5,6,7,8** | *Understand independence and conditional probability and use them to interpret data*<br><br>S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).<br><br>S.CP.2 Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.<br><br>S.CP.3 Understand the conditional probability of $A$ given $B$ as $P(A \text{ and } B)/P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.<br><br>S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*<br><br>S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* |

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| | *Use the rules of probability to compute probabilities of compound events in a uniform probability model*<br><br>S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.<br><br>S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.<br><br>*Use probabilities to evaluate outcomes of decisions*<br><br>S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).<br><br>S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). |
| **Unit 5:**<br><br>**Right Triangle Trigonometry & Pythagorean Theorem**<br>(30 days)<br><br>**SMP-1,2,3,4,5,6,7,8** | *Prove theorems involving similarity*<br><br>G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*<br><br>*Define trigonometric ratios and solve problems involving right triangles*<br><br>G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.<br><br>G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. |

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| | G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ |
| **Unit 6:**<br><br>**Extending to Three Dimensions**<br>(15 days)<br><br>**SMP-1,2,3,4,5,6,7,8** | *Apply geometric concepts in modeling situations*<br><br>G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★<br><br>G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★<br><br>G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★<br><br>*Explain volume formulas and use them to solve problems*<br><br>G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* ***Not assessed on WA State EOC***<br><br>G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★<br><br>*Visualize relationships between two-dimensional and three-dimensional objects*<br><br>G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. |

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| **Unit 7:**<br><br>**Connecting Algebra and Geometry through Coordinates**<br>(15 days)<br><br>**SMP-1,2,3,4,5,6,7,8** | *Use coordinates to prove simple geometric theorems algebraically*<br><br>G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *Triangles/Quads check notes*<br><br>G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).<br><br>G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.<br><br>G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ |
| **Unit 8:**<br><br>**Circles with and without Coordinates**<br>(15 days)<br><br>**SMP-1,2,3,4,5,6,7,8** | *Understand and apply theorems about circles*<br><br>G.C.1. Prove that all circles are similar.<br><br>G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*<br><br>G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. |

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| | *Find arc lengths and areas of sectors of circles*<br><br>G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.<br><br>*Translate between the geometric description and the equation for a conic section*<br><br>G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.<br><br>*Use coordinates to prove simple geometric theorems algebraically*<br><br>G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* |